C24 – AIR CAVITY STACK EFFECT

1. Introduction

An empirical method is used to calculate the stack effect in an air cavity or in a room that is connected to an environment at a different temperature through an inlet and an outlet at different levels. The method is implemented in the spreadsheet C24_air_cavity_stack_effect.xlsm and can be used to generate ventilation functions for BISTRA and VOLTRA.

2. Geometry

Figure 1 below shows a cavity scheme. The rectangular air cavity (if D is small) or room (if D is large) with length L is connected to an external environment through an inlet at the bottom and an outlet at the top. The cavity mean temperature is θ_i, the environmental temperature is θ_e.

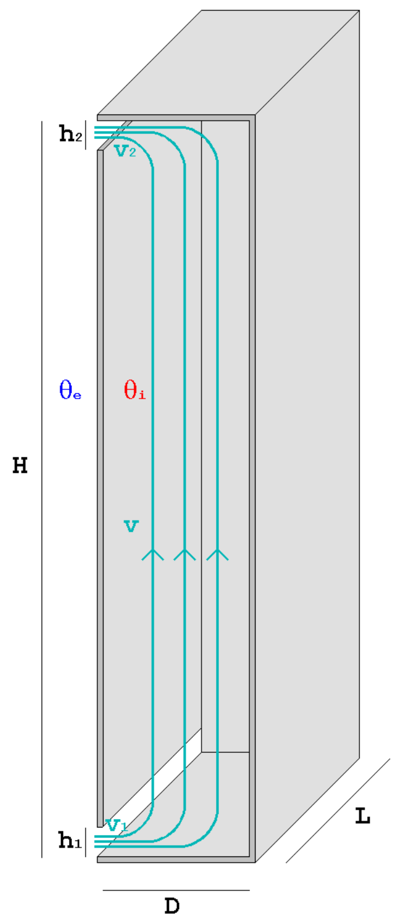

Figure 1 Cavity scheme

3. Calculation method

The pressure difference causing the stack effect

As the air density depends on the air temperature, the temperature difference between the cavity and the external environment causes a pressure difference.

$$\Delta p = \rho_e \cdot \frac{T_i - T_e}{T_i} \cdot g \cdot H \quad (1)$$

with:			
	Δp	[Pa]	pressure difference
	ρ_e	[kg/m³]	external air density
	T_i	[K]	mean cavity absolute temperature
	T_i	[K]	external absolute temperature
	g	[m²/s]	gravitational acceleration
	H	[m]	cavity height

The pressure difference causes an air movement. The air movement causes friction losses. Both effects have to be in equilibrium.

Reynolds number

The airflow can be laminar or turbulent depending on the Reynolds number Re.

$$Re = \frac{v \cdot D_h}{\nu} \quad (2)$$

with:			
	Re	[-]	Reynolds number
	v	[m/s]	air velocity
	D_h	[m]	hydraulic diameter $\quad D_h = \frac{4DL}{2(D+L)} \quad (3)$
	ν	[m²/s]	kinematic viscosity of air

For $R_e \leq 2500$ the airflow is laminar, for $R_e > 2500$ the airflow is turbulent.

Friction losses

The friction losses are caused by the walls bordering the cavity and by the inlet and outlet.

The friction losses caused by the cavity walls are calculated using:

$$\Delta p_c = f \cdot \frac{H}{D_h} \cdot \frac{\rho \cdot v^2}{2} \quad (4)$$

with:			
	Δp_c	[Pa]	pressure difference over the cavity
	H	[m]	cavity height
	D_h	[m]	hydraulic diameter
	ρ	[kg/m³]	air density
	v	[m/s]	air velocity
	f	[-]	friction factor

for laminar airflow $\quad f = \frac{64}{Re} \quad (5)$

for turbulent flow with $Re < 10^5$ $\quad f = \frac{0.3164}{Re^{0.25}} \quad (6)$

for turbulent flow with $10^5 <= Re < 3 \cdot 10^6$ $$f = 0.0032 + \frac{0.221}{Re^{0.237}} \quad (7)$$

The friction losses of the inlet and outlet are calculated using:

$$\Delta p_i = \xi_i \cdot \frac{\rho \cdot v_i^2}{2} \quad (8)$$

with:			
	Δp_i	[Pa]	pressure difference for the local resistance i (1 or 2)
	ξ_i	[-]	friction factor of the local resistance i: inlet ξ = 0.5 / outlet ξ = 1.0
	ρ	[kg/m³]	air density
	v_i	[m/s]	local air velocity $\quad v_i = \frac{v \cdot D}{h_i} \quad$ (9)
	v	[m/s]	air velocity in the cavity
	D	[m]	cavity width
	h_i	[m]	height of inlet (i = 1) or outlet (i = 2)

The total pressure difference due to the friction losses in inlet, cavity and outlet is:

$$\Delta p = \xi_1 \cdot \frac{\rho \cdot v_1^2}{2} + f \cdot \frac{H}{D_h} \cdot \frac{\rho \cdot v^2}{2} + \xi_2 \cdot \frac{\rho \cdot v_2^2}{2} \quad (10)$$

Solution

From the equilibrium between the pressure difference causing the stack effect and pressure difference due to the friction losses, the air velocity in the cavity can be derived.

4. Derived quantities

Airflow Q_m

$$Q_m = v\,D\,L \quad (11)$$

with:			
	Q_m	[m³/s]	airflow
	v	[m/s]	air velocity in the cavity
	D, L	[m]	cavity width, length

Ventilation rate n

$$n = \frac{v \cdot 3600}{H} \quad (12)$$

with:			
	n	[/h]	ventilation rate (volumes per hour),
	v	[m/s]	air velocity in the cavity
	H	[m]	cavity height

Airflow Q_m as a function of temperature difference

For a given cavity the airflow Q_m can be expressed as a function of the temperature difference between cavity and environment.

$$Q_m = b \cdot |\Delta\theta|^{0.5} \quad (13)$$

with:			
	Q_m	[m³/s]	airflow
	$\Delta\theta$	[K]	temperature difference between cavity and environment
	b	[m³/sK$^{0.5}$]	coefficient depending on θ_e, H, D, L, ξ_1 and ξ_2.

Enthalpy flow Q

The enthalpy flow is the difference in heat content between the air leaving the cavity and the one entering the cavity. 2 cases are distinguished:

1) Linear cavity air temperature gradient (in case of cavity air heated along its trajectory).

$$Q_{T_{linear}} = \rho \; c \; Q_m 2(T_i - T_e) \quad (14)$$

2) Homogeneous air temperature (in case of cavity heated at the bottom).

$$Q_{T_{homogeneous}} = \rho \; c \; Q_m(T_i - T_e) \quad (15)$$

with:

with:	$Q_{Tlinear}$	[W]	enthalpy flow in case of a linear cavity air temperature gradient
	ρ	[kg/m^3]	air density (at mean cavity temperature)
	c	[J/kgK]	mass heat capacity of air
	Q_m	[m^3/s]	airflow
	T_i	[K]	mean cavity absolute temperature
	T_e	[K]	external absolute temperature.

5. Example

The equations are implemented in the Excel file with C24 air cavity stack effect.xlsm, which contains VBA macros (To use this spreadsheet correctly, the Excel Solver add-in must be installed, and macros/content must be enabled)

Figure 2 displays the layout of the spreadsheet. Green cells indicate user inputs, orange cells shows the computed results, and light blue cells contain physical and standard constants.

The results obtained from this spreadsheet can be applied to various scenarios.

For example, in Section 5 of the VOLTRA Tutorial Part B (Physibel Knowledge Base: T-VTRb - Transient simulations of ventilated double skin façade using the solar processor: ventilation flows and control functions), the airflow rate is used as a function of the temperature difference. The coefficient required for that calculation is derived using the Excel sheet referenced in this document.

environmental temperature	$\theta_e =$	20,0	°C
cavity temperature	$\theta_i =$	40,0	°C
cavity height	$H =$	3,000	m
cavity width	$D =$	0,300	m
cavity length	$L =$	1,000	m
cavity section	$A_c =$	0,300	m^2
cavity volume	$V_c =$	0,900	m^3
inlet height	$h_1 =$	0,010	m
inlet surface	$A_1 =$	0,010	m^2
inlet local friction factor	$\xi_1 =$	0,5	-
outlet height	$h_2 =$	0,010	m
outlet surface	$A_2 =$	0,010	m^2
outlet local friction factor	$\xi_2 =$	1,0	-
air properties			
environmental air density	$\rho_e =$	1,20	kg/m^3
cavity air density	$\rho_i =$	1,13	kg/m^3
cavity air dynamic viscosity	$\mu_i =$	1,92E-05	Pa.s
cavity air kinematic viscosity	$\nu_i =$	1,70E-05	m^2/s
cavity air specific heat	$c_{pi} =$	1008	J/kg.K
cavity air thermal conductivity	$\lambda_i =$	0,027	W/m.K
cavity air thermal diffusivity	$\alpha_i =$	2,393E-05	m^2/s
Prandtl number	$Pr =$	0,712	-
air flow results			
hydraulic diameter	$D_h =$	0,462	m
pressure difference caused by density difference	$\Delta p =$	2,26	Pa
pressure difference caused by friction losses	$\Delta p =$	2,26	Pa
air velocity in the cavity	$v =$	0,05	m/s
air velocity in the inlet	$v_1 =$	1,64	m/s
air velocity in the outlet	$v_2 =$	1,64	m/s
air flow	$Q_m =$	0,016	m^3/s
air flow	$Q_m =$	58,90	m^3/h
coefficient b in $Q_m = b * \|\Delta\theta\|^{0.5}$	$b =$	0,00366	$m^3/sK^{0.5}$
ventilation rate	$n =$	65,44	/h
heat flow in case of linear temperature gradient in cavity	$Q_{Tlinear} =$	743	W
heat flow in case of homogeneous temperature in cavity	$Q_{Thomogenoeus} =$	372	W
friction factor (ASHRAE Fundamentals 1997, 2.8)	$f =$	0,0433	-
Reynolds number	$R_{e,Dh} =$	1478	-
flow type		laminar	
constants used for calculating thermophysical air properties			
standard atmospheric pressure	$P =$	101300	Pa
gravity acceleration	$g =$	9,81	m/s^2
universal gas constant	$R =$	8,314472	J/K.mol
air molecular mass	$M =$	0,02897	kg/mol
absolute temperature for °C	$T_0 =$	273,15	K
ISO 15099 table B1 constant a	$\lambda a =$	2,873E-03	W/m.K
ISO 15099 table B1 constant b	$\lambda b =$	7,760E-05	W/m.K
ISO 15099 table B2 constant a	$\mu a =$	3,723E-06	Pa.s
ISO 15099 table B2 constant b	$\mu b =$	4,940E-08	Pa.s
ISO 15099 table B3 constant a	$c_p a =$	1003,737	J/kg.K
ISO 15099 table B3 constant b	$c_p b =$	0,012324	J/kg.K

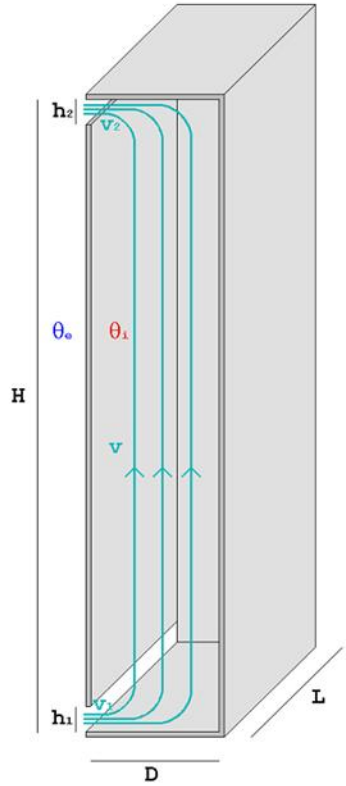

Figure 2 Excel sheet for stack effect